

MATH 370 - Computational algebra

LAB #3 - Leading Terms

1) How do loops work in CoCoA? Understand the difference between For, ForEach, While and Repeat using the solution of LAB 2

2) Learn how to change the "active" term ordering in CoCoA. Try Lex, DegLex, and DegRevLex.

3) Calculate with CoCoA the leading term of the following polynomials with respect to the three term orderings we studied. To do this, you need to redefine the ring every time you change the term ordering. Check that your results are consistent with the definition of Lex, DegLex and DegRevLex we saw in class.

$$f = x^2y^4z^5 - 5x^3yz^7 - y^3z^3 - 2x^4z \quad \mathcal{Q} \ x, y, z$$

$$g = x_1^2x_2^5x_4^3 - x_1^5x_2x_3x_4^3 \quad \mathcal{Q} \ x_1, x_2, x_3, x_4$$

4) Let \prec be any term ordering. Define the following "weak" order on polynomials: we say that

$$f \prec g \text{ if } LT(f) \prec LT(g)$$

Prove that this relation does not respect axiom A_2 : find two polynomials f and g such that $f \prec g$ and $f \prec g$, but $f \not\prec g$. If you work in one variable it's easier.

5) Learn how to use the If...Then...Else syntax in CoCoA. See the manual.

6) Write a CoCoA function which takes two polynomials and returns the greatest one with respect to the weak order defined in 4). If $f \prec g$ and $f \prec g$, then return both (as a list of two elements). Apply your function to the following two polynomials of $\mathcal{Q} \ x, y, Lex$:

$$f = x^2y - y^2 \quad g = x^2y - 1$$

7) Write a CoCoA function which takes a polynomial and returns its support (i.e. the list of its nonzero terms), ordered with respect to the active term ordering, from the lowest term to the highest one.